



Random subspace based ensemble sparse representation



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ABSTRACT

In this paper, a new random subspace based ensemble sparse representation (RS_ESR) algorithm is proposed, where the random subspace is introduced into sparse representation model. For high-dimensional data, the random subspace method can not only reduce dimension of data but also make full use of effective information of data. It is not like traditional dimensionality reduction methods that may lose some information of original data. Additionally, a joint sparse representation model is employed to obtain the sparse representation of a sample set in the low dimensional random subspace. Then the sparse representations in multiple random subspaces are integrated as an ensemble sparse representation. Moreover, the obtained RS_ESR is applied in classical clustering and semi-supervised classification. The experimental results on different real-world data sets show the superiority of RS_ESR over traditional methods.

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1. Introduction

A large amount of data is easily obtained with the ever-accelerated updating of information technology. As an effective technique for analyzing the sparsity of large data, sparse representation (SR) [1–2] method emerges as the times require. In recent years, SR has attracted attention of many researchers and been successfully applied in image classification and other fields, such as signal reconstruction [3], image super-resolution [4], visual tracking [5], face recognition [6–7], etc. At first, Wright et al. [8] proposed a sparse model-based method for facial images classification. During the same time period, a sparse subspace clustering algorithm was presented by Elhamifar et al. [9], which combines the SR technique with spectral clustering to segment different moving objects in the video. Hereafter, many improved sparse representation based classification methods keep emerging [10–17].

Although sparse representation based classification methods are very effective, low memory problem occurs when sparse representation is used to deal with high dimensional data. Since sparse representation method updates the similarity between every two samples in each iteration, it needs massive computation and storage, especially for the high dimensional dataset. In fact, the data usually lies in a high dimensional space in many real applications. It is well known that an effective method to handle the high dimensional data is dimensionality reduction. The common dimensionality

reduction approaches include principal component analysis [18], linear discriminant analysis [19], locality preserving projections [20], etc. After the rise of SR, a new dimensionality reduction method called sparsity preserving projections (SPP) was proposed [21]. But whatever dimensionality reduction approach is used, the reduction of the dimension of data in the original space results in information loss. In this case, the spatial relationship among the samples in lower dimensional space may be changed, which affects the following clustering or classification results.

To make full use of potential information, a random subspace method is used in this paper. The main difference between it with the traditional dimensionality reduction approaches is that the random subspace method randomly samples many lower-dimensional subspaces from the original high-dimensional space. The random subspace method has been successfully applied in classifier ensemble [22–23], which is more robust to noise and redundant information than a single classifier. This indicates that several random lower-dimensional subspaces include more effective information than the original high-dimensional space. The success of the random subspace method in classifier ensemble motivates us to apply it to reduce dimension.

In this paper, we combine the SR with the random subspace, and propose a new algorithm called random subspace based ensemble sparse representation (RS_ESR). Firstly, multiple subspaces are obtained from the original high-dimensional space by using the random subspace method. Then a joint sparse representation model is used to simultaneously calculate the sparse representation of samples in each subspace. Afterwards, the sparse repre-

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sentations in all lower-dimensional subspaces are integrated into an ensemble sparse representation. Finally, the proposed RS_ESR is applied in dataset clustering, image segmentation and semi-supervised classification.

The remaining parts of this paper are organized as follows. Section 2 reviews the classical SR model and random subspace method. The random subspace based joint SR model and RS_ESR method are described in Section 3. Section 4 presents the RS_ESR based clustering and semi-supervised classification algorithms. Experimental results in different datasets and images are contained in Section 5. Finally, Section 6 provides conclusions and discussion of possible improvements in future work.

2. Related work

2.1. Sparse representation (SR)

The fundamental of SR is that any one $\mathbf{x} \in \mathbb{R}^d$ of a dataset can be represented by a linear combination of bases in a dictionary $D \in \mathbb{R}^{d \times l}$ ($d \ll l$). The weights of all atoms in the linear combination are sparse and named as SR. The SR $\mathbf{z}$ of sample $\mathbf{x}$ can be obtained by solving the following model [1]

$$\min_{\mathbf{z}} \|\mathbf{z}\|_0 \text{ s.t. } \mathbf{x} = D\mathbf{z}, \quad (1)$$

where $\|\cdot\|_0$ is ℓ_0 norm of a vector. The objective function in (1) is an NP-hard problem, so it is replaced with

$$\min_{\mathbf{z}} \|\mathbf{z}\|_1 \text{ s.t. } \mathbf{x} = D\mathbf{z}, \quad (2)$$

where $\|\cdot\|_1$ is ℓ_1 norm of a vector.

2.2. Random subspace

The random subspace [24] method was first presented in decision forest, in which multiple decision trees were generated in multiple random subspaces and then integrated into a classifier. On that basis, random forest [25] and rotation forest [26] were consecutively put forward. The method to get the random subspace includes two steps, as detailed in Algorithm 1.

One possible problem is that multiple lower-dimensional datasets obtained by the random subspace method may not include the discriminative information of original dataset. In fact, it seldom happens. The larger q and p , the smaller the probability that no discriminative information is selected in q random subspaces is. The specific reasons have been analyzed in [24] and [27], so we will not cover them in this paper.

3. Random subspace based ensemble sparse representation (RS_ESR)

3.1. Random subspace based ensemble sparse representation

A proposed multi-task low-rank affinity pursuit (MLAP) [28] algorithm integrated multiple types of features and effectively used cross-feature information of multiple features. Inspired by the MLAP algorithm, we consolidate multiple random subspace sets in a similar way. But a great deal of computer's internal storage and computer time are spent to solve the nuclear norm of the matrix in MLAP. Thus, to improve the efficiency of algorithm, we introduce a novel random subspace based joint sparse representation model

$$\min_{\substack{Z^1, \dots, Z^q \\ S^1, \dots, S^q}} \sum_{t=1}^q \left(\|Z^t\|_1 + \alpha \|S^t - S^t L^t\|_F^2 \right) + \beta \|Z\|_{2,1} \quad (4)$$

$$\text{s.t. } S^t = X(\mathbf{r}^t), \text{diag}(Z^t) = 0,$$

where X denotes the data set in original space; S^t is t th random subspace set of X obtained by Algorithm 1; Z^t is sparse representation of S^t , and $\|Z^t\|_1 = \sum_{i,j} |Z_{ij}^t|$; $\|\cdot\|_F$ is Frobenius norm; $\alpha > 0$ and $\beta > 0$ are parameters to balance the effect of different parts; the $n^2 \times q$ matrix Z is constructed in the same way as the MLAP algorithm; $\|Z\|_{2,1} = \sum_i \sqrt{(\sum_j |Z_{ij}|)^2}$. The constraint $\text{diag}(Z^t) = 0$ is to avoid that the solution of (4) is the identity matrix.

We employ the inexact augmented Lagrange multiplier method, also called alternating direction method of multipliers (ADMM) [29] to solve (4). Firstly, we introduce two auxiliary variables K^t and L^t , so the objective function in (4) is converted into equivalent form as follows:

$$\min_{\substack{K^1, \dots, K^q \\ L^1, \dots, L^q \\ Z^1, \dots, Z^q}} \sum_{t=1}^q \left(\|K^t\|_1 + \alpha \|S^t - S^t L^t\|_F^2 \right) + \beta \|Z\|_{2,1}$$

$$\text{s.t. } Z^t = K^t, \text{diag}(K^t) = 0, Z^t = L^t$$

Then minimize the following augmented Lagrange function

$$\beta \|Z\|_{2,1} + \sum_{t=1}^q \left(\|K^t\|_1 + \alpha \|S^t - S^t L^t\|_F^2 \right) + \sum_{t=1}^q \left(\langle U^t, Z^t - K^t \rangle + \langle V^t, Z^t - L^t \rangle + \frac{\mu}{2} \|Z^t - K^t\|_F^2 + \frac{\mu}{2} \|Z^t - L^t\|_F^2 \right)$$

where $U^1, \dots, U^q$ and $V^1, \dots, V^q$ are Lagrange multipliers and $\mu > 0$ is a penalty parameter. By the ADMM method, the problem (4) is divided into several sub-problems which have closed-form solutions. The solution procedure of (4) is detailed in Algorithm 2.

In Algorithm 2, we utilize the soft thresholding operator [30] to solve (5) and the objective function in (6) can be solved according to Lemma 3.2 in [31].

Let $\tilde{Z}^1, \dots, \tilde{Z}^t, \dots, \tilde{Z}^q$ be the optimal solution of the problem (4). These sparse representations $\tilde{Z}^1, \dots, \tilde{Z}^t, \dots, \tilde{Z}^q$ in random subspaces are integrated by

$$E_{i i'} = \sqrt{\sum_{t=1}^q (\tilde{Z}_{i i'}^t)^2}, \quad (7)$$

where $\tilde{Z}_{i i'}^t$ ($1 \leq i \leq n, 1 \leq i' \leq n$) and $E_{i i'}$ are the i' th element in i th row of $\tilde{Z}^t$ and E respectively, and E is viewed as ensemble sparse representation.

From the methodology point of view, the obtained ensemble sparse representation can be directly used for existing clustering and classification methods based on sparse representation. On the other side, by integrating the multiple sparse representations in subspaces, effective information in multiple lower-dimensional datasets is reinforced and redundant information is weakened. So the integrated ensemble sparse representation is more helpful for the clustering and classification than the sparse representations in subspaces. Algorithm 3 summarizes the generation process of the ensemble sparse representation E . It reflects the similarity among samples in the original dataset X . The larger the ensemble sparse representation coefficient $E_{i i'}$ is, the more similar the corresponding i th and i' th samples are, and vice versa.

3.2. Computational complexity analysis

The computational complexity of RS_ESR is analyzed in this subsection. We suppose that the number of iterations is e . The time complexity of the proposed RS_ESR algorithm is $O(n^2 p q e)$. In fact, if the random subspace method is not used for RS_ESR, the RS_ESR algorithm is equivalent to the existing L1-graph [32]. The computing complexity of L1-graph is $O(n^2 d e)$. Generally, the initial dimension of sample d is close to $p q$. So the time complexities of RS_ESR and L1 are roughly equivalent. Furthermore, since

MLAP used singular value thresholding operator to solve the low-rank problem of it, the computational complexity of random subspace based MLAP (RS_MLAP) is $O(n^3pqe)$. It is obviously higher than RS_ESR.

4. Image analysis with RS_ESR

An effective sparse representation is critical for the SR based learning algorithms. Many existing SR models were successfully applied in various tasks, such as reconstruction, clustering, embedding, classification, etc. In this section, we briefly introduce how to combine RS_ESR with spectral clustering and semi-supervised learning algorithms.

4.1. Spectral clustering with RS_ESR

Spectral clustering algorithm [33] is one of the most popular clustering methods and has been widely used [34–35] in recent years. It is simple to implement, as there is only one parameter to control the similarity between a data pair. Treating the ensemble sparse representation E obtained by RS_ESR as similarity matrix, RS_ESR based spectral clustering algorithm is precisely described in Algorithm 4.

4.2. Semi-supervised classification based on RS_ESR

In recent years, semi-supervised learning [36] has attracted attentions of many researchers [37–38]. It is because the semi-supervised classification method always gets a favorable classification just by using a small amount of label information. Known label information can be propagated to other unlabeled samples through the graph constructed by the RS_ESR coefficients. Algorithm 5 gives the brief process of RS_ESR based semi-supervised classification algorithm.

5. Experimental results and analysis

In this section, we evaluated the performance of the proposed RS_ESR in spectral clustering and semi-supervised classification. RS_ESR is compared with other state-of-the-art algorithms: k-nearest neighbors (KNN) [39], locally linear embedding (LLE) [40], sparsity preserving projections (SPP), L1-graph (L1), and random subspace based MLAP (RS_MLAP). In the experiment, the six algorithms are considered as a method of feature extraction, and then the clustering and classification results are obtained by Algorithms 4 and 5 respectively. Besides, the experimental results of RS_ESR are also compared with that of other common clustering methods, including K-means, mean shift (MS), fuzzy c-means (FCM) and spectral clustering (SC) in the data clustering experiment. In addition, the improved algorithms of MS, FCM, SC and a new method are compared with our proposed approach in the image segmentation experiment. The experimental results show that the proposed RS_ESR in this paper has better data adaptiveness and noise robustness than other methods.

5.1. Clustering with UCI dataset

To objectively evaluate the clustering performances of K-means, mean shift, FCM, spectral clustering, KNN, LLE, SPP, L1, RS_MLAP and RS_ESR on four UCI benchmark clustering datasets, we use three estimation indexes: clustering accuracy (CA), rand index (RI) [41], and normalized mutual information (NMI) [42]. Table 1 shows brief descriptions of four datasets, and the numbers of feature of them are large.

As you know, the initial clustering centers and degree of membership of clustering algorithms are random, thus each method

Table 1

Descriptions of four datasets showing the number of samples, the number of features, and the number of classes for each dataset.

Dataset	Number of samples	Number of features	Number of clusters
Wine	178	13	3
ForestTypes	326	27	4
Ionosphere	351	34	2
Faults	1941	27	7

was performed twenty times and the mean value of these results was shown in Table 1. In our experiments, the bandwidth parameters of Wine, ForestTypes, Ionosphere, and Faults datasets are respectively set to 180, 49.5, 5.25, and 1,200,000 in mean shift [43] method to make the number of classes of clustering results equivalent to the ideal value in Table 1. The weighting exponent parameter of FCM is set to 2. To obtain the best clustering results, the Gaussian parameters of Wine, ForestTypes, Ionosphere, and Faults datasets are respectively set to 0.01, 0.5, 3, and 0.1 in the spectral clustering [44] algorithm. Similarly, the number of adjacent samples in the KNN and LLE algorithms is set to 10. The parameters of RS_ESR have also been adjusted to be the best with $\alpha = 0.5$ and $\beta = 0.5$ in (4). In this subsection, we set the number of random subspace q to 3 and dimensionality of each random subspace p to 10.

The clustering results and running time of K-means, mean shift, FCM, spectral clustering, KNN, LLE, SPP, L1, RS_MLAP and RS_ESR on four UCI datasets are shown in Table 2. It can be easily seen from Table 2 that the clustering results of the proposed RS_ESR are better than that of other algorithms, except that the experimental result of RS_ESR on the ForestTypes dataset is slightly below that of K-means and FCM. But the difference between the clustering accuracy of RS_ESR with that of K-means and FCM on the ForestTypes dataset is only in (e^{-2}, e^{-1}) . Moreover, the performance of RS_ESR in other datasets is better than K-means and FCM. It means that the performance of the proposed RS_ESR algorithm is favorable and stable for different datasets.

In Addition, these methods are run on a personal computer with 3.20 GHz Intel(R) Core(TM) i3 processors, 3 GB memory and Windows XP operating system, using programs written by MATLAB R2010a. Although RS_ESR runs slowly than KNN, LLE, SPP, and L1, the runtime of RS_ESR is lower than RS_MLAP, as shown in Table 2.

5.2. Segmentation of artificial images

To further estimate the performance of RS_ESR in image segmentation, a series of experiments on artificial images are implemented. By using the corresponding ground truth, the segmentation results can be directly appraised by the segmentation accuracy (SA) and runtime. Similarly, we performed twenty times for each method, and retained the mean value of their segmentation accuracies which corresponding segmentation result is showed.

To be fair, an improved mean shift method [45] and a fuzzy local information c-means (FLICM) [46] algorithm with the highest citation are employed in the image segmentation experiments. For different images, the parameters (h_s, h_r, W) of mean shift are adjusted to make the visual effects and accuracy of segmentation results to be the best. Additionally, the proposed algorithm is compared with two new approaches: an efficient Markov random field embedded level set (MRF+LS) method [47] and iterative ensemble normalized cuts (IENCut) [48] in spectral clustering. In the MRF+LS method, parameters λ , γ , and Δt are set to 0.65, 0.25, and 30, respectively. As the segmentation result of MRF+LS is boundary image, each region of the segmentation result is artificially labeled according to the original image for comparison purposes. The parameters of the IENCut algorithm are set as follows:

Table 2
Clustering results and runtime (second) of six algorithms for different datasets.

		K-means	MS	FCM	SC	KNN	LLE	SPP	L1	RS_MLAP	RS_ESR
Wine	CA	0.7022	0.5899	0.6708	0.4157	0.6292	0.7079	0.5618	0.8989	0.9045	0.9157
	RI	0.7187	0.5857	0.7331	0.5606	0.6393	0.7191	0.618	0.8725	0.8773	0.8913
	NMI	0.4288	0.388	0.443	0.0217	0.3532	0.4193	0.1691	0.7309	0.7125	0.739
	Time	0.101	28.042	0.0721	0.1117	0.0084	0.0106	1.3763	1.1136	38.7856	20.2404
ForestTypes	CA	0.6954	0.4308	0.6708	0.2954	0.6062	0.5969	0.4246	0.3846	0.5754	0.6215
	RI	0.7271	0.3356	0.7331	0.5954	0.6885	0.6863	0.3202	0.6223	0.602	0.6946
	NMI	0.4348	0.0439	0.443	0.0149	0.3455	0.3565	0.0206	0.1249	0.3186	0.3699
	Time	0.0882	5.4219	0.0721	0.1638	0.019	0.0214	4.6694	4.4224	197.1524	90.7189
Ionosphere	CA	0.7123	0.6439	0.7094	0.7094	0.6268	0.6381	0.6382	0.5413	0.6439	0.7179
	RI	0.5889	0.5401	0.5865	0.5865	0.5308	0.5387	0.5369	0.502	0.5401	0.5938
	NMI	0.1349	0.0087	0.1299	0.1299	0.0825	0.0481	0.0038	0.1195	0.0087	0.15
	Time	0.1546	0.0351	0.0889	0.2033	0.0172	0.0433	0.4929	0.4811	251.9357	108.927
Faults	CA	0.2978	0.3488	0.2834	0.2885	0.3462	0.3349	0.304	0.3081	0.3745	0.4152
	RI	0.6753	0.2482	0.6695	0.516	0.5859	0.6467	0.3542	0.4528	0.5933	0.6513
	NMI	0.1037	0.0227	0.0627	0.0058	0.0589	0.0918	0.0154	0.0123	0.0879	0.1402
	Time	0.1293	9.5281	0.35	2.7284	0.2219	0.2794	7.6466	7.1036	2316.4563	998.149

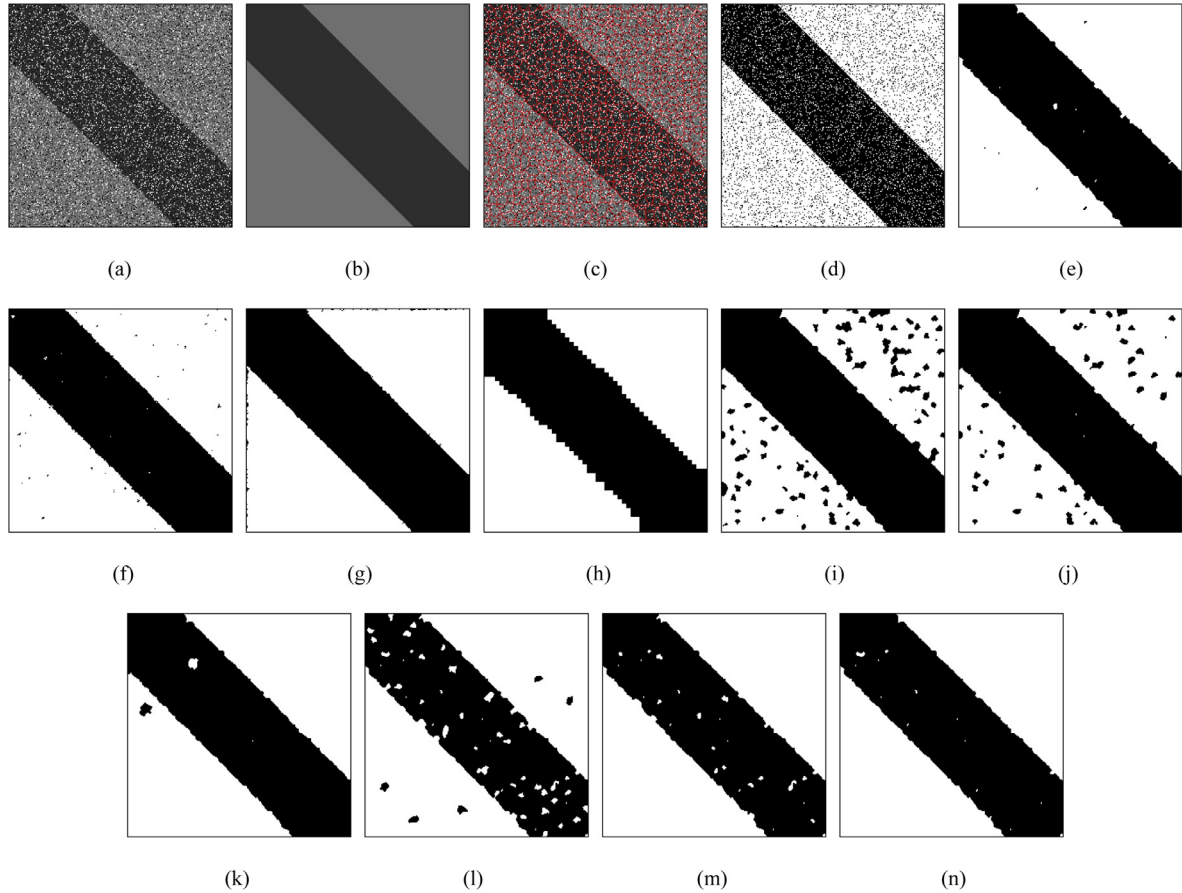


Fig. 1. Segmentation results on the 2-class artificial image. (a) Artificial image, (b) ground truth, (c) superpixels obtained by Turbopixels algorithm, (d) K-means, (e) Mean shift, (f) FLICM, (g) MRF+LS, (h) IENCut, (i) KNN, (j) LLE, (k) SPP, (l) L1, (m) RS_MLAP, (n) RS_ESR.

Gaussian kernel parameter $\sigma = 0.5$, groups size $N_g = 80$, validation data size $N_{val} = 500$, gap-normalized distance threshold $thr = 0.5$. IENCut first downsamples each input image every 5 pixels, resulting in a smaller output image. The segmentation result of IENCut is upsampled as 5 times for viewing purposes in this paper. The following segmentations of MRF+LS and IENCut are processed by the same way. Similarly, the number of adjacent samples in the KNN and LLE methods is set to 10. Besides, in the experiments of image segmentation, the number of random subspace q and the dimensionality of each random subspace p are set to 10 and 30 respectively.

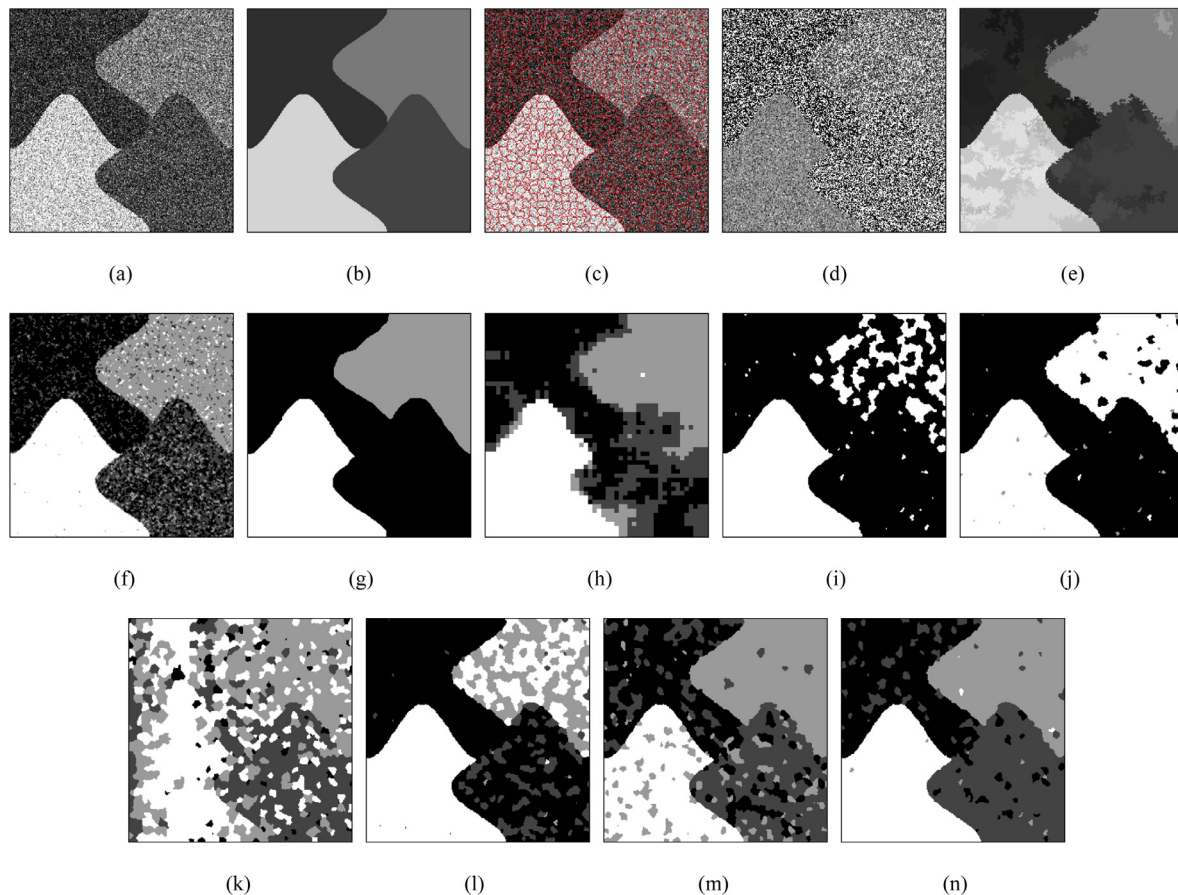
Moreover, since the number of image pixels is large, we firstly utilize Turbopixels [49] method to get superpixels for the KNN, LLE, SPP, L1, RS_MLAP and RS_ESR methods, and set the number of superpixels in Turbopixels algorithm to 800. Then the gray histogram and mean value of each superpixel are extracted to form a 257-dimensional feature. Finally, the feature set of all superpixels is clustered to obtain the segmentation results.

The first artificial image is a 2-class noise image with the size of 256×256 shown in Fig. 1(a), which is obtained by artificially adding salt and pepper noise 20% to the corresponding 2-class ground truth image. The 2-class ground truth image has two gray levels 60 and 130, as shown in Fig. 1(b). Fig. 1(c) shows the super-

Table 3

Segmentation accuracies and runtime (second) of different algorithms on two artificial images.

		K-means	MS	FLICM	MRF + LS	IENCut	KNN	LLE	SPP	L1	RS_MLAP	RS_ESR
Fig. 1(a)	SA	0.9013	0.9933	0.9935	0.9905	0.9848	0.9409	0.9688	0.9909	0.9667	0.988	0.9937
	Time	0.07	4.84	2.6628	2.13	0.5604	0.0706	0.0673	91.863	89.029	396.3898	222.5491
Fig. 2(a)	SA	0.5046	0.305	0.7994	0.732	0.8114	0.5009	0.5021	0.5417	0.6732	0.8192	0.9325
	Time	0.2538	5.12	11.984	2.86	0.4594	0.0679	0.119	83.898	84.859	622.8472	187.1894

**Fig. 2.** Segmentation results on the 4-class artificial image. (a) Artificial image, (b) ground truth, (c) superpixels obtained by Turbopixels algorithm, (d) K-means, (e) Mean shift, (f) FLICM, (g) MRF+LS, (h) IENCut, (i) KNN, (j) LLE, (k) SPP, (l) L1, (m) RS_MLAP, (n) RS_ESR.

pixels of Fig. 1(a) obtained by Turbopixels algorithm. The segmentation results of K-means, KNN, LLE, and L1 have many spots in consistent regions, which are shown in Fig. 1(d), (i), (j), and (l) respectively. Fig. 1(e), (f), (k), (m), and (n), obtained respectively by mean shift (with $(h_s, h_r, W) = (7, 6.5, 20)$), FLICM, SPP, RS_MLAP, and RS_ESR, improve the regional consistency. Although Fig. 1(g) and (h) obtained by MRF+LS and IENCut respectively have best regional consistency, many pixels are misclassified in the boundaries of Fig. 1(g) and (h). The corresponding segmentation accuracies of these methods are shown in Table 3. As can be seen from Table 3, the segmentation accuracy of RS_ESR is highest.

Similarly with Fig. 1(a), Fig. 2(a) is generated by adding Gaussian noise 20% to a 256×256 4-class image (which has four gray levels 60, 85, 140 and 220) shown in Fig. 2(b). Likewise, the superpixels of Fig. 2(a) obtained by Turbopixels algorithm are shown in Fig. 2(c). Fig. 2(d), (i)–(k) show respectively the segmentation of K-means, KNN, LLE and SSP, which confuse the pixels from different classes. Although the segmentation of mean shift with $(h_s, h_r, W) = (13, 7, 300)$ can separate the pixels of different classes, it contains 239 categories, as shown in Fig. 2(e). The seg-

mentation result of MRF+LS shown in Fig. 2(g) has good regional consistency, but the pixels belonging to two classes are confused. Although the segmentations of IENCut and L1 are similar with MRF+LS, they have bad regional consistency, as shown in Fig. 2(h) and (l) respectively. Fig. 2(f) and (m) show the segmentation results of FLICM and RS_MLAP respectively, which have many spots in consistent regions. In contrast, Fig. 2(n) obtained by RS_ESR has better regional consistency than Fig. 2(f) and (m). The accuracies of segmentation corresponding to Fig. 2 are shown in Table 3.

Furthermore, the runtime of different algorithms on Figs. 1(a) and 2(a) is also shown in Table 3. In order to make the results in Table 3 more intuitive, the histograms of segmentation accuracies and runtime of different algorithms on Figs. 1(a) and 2(a) are shown in Figs. 3 and 4 respectively. From Table 3 and Fig. 3, you can see that the performance of RS_ESR outperforms L1, which means that the random subspace method is effective. Meanwhile, as can be seen from Table 3 and Fig. 4, though the runtime of RS_ESR is longer than that of K-means, mean shift, FCM, spectral clustering, KNN, LLE, SPP, and L1, RS_ESR runs quicker than RS_MLAP.

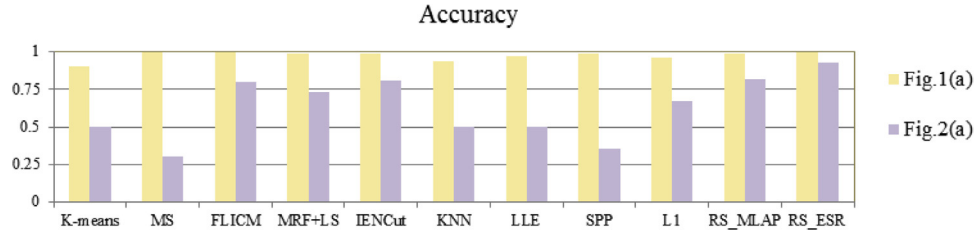


Fig. 3. The accuracy rates of the segmentations obtained by different methods for two noise images.

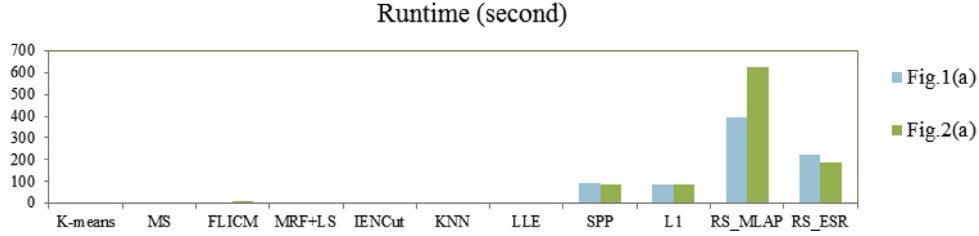


Fig. 4. The runtime (second) of different methods for two noise images.

5.3. Segmentation of SAR images

To validate the applicability of RS_ESR, the experiments on two real SAR images are implemented. As the ground truths of SAR images are generally absent, the evaluation of the segmentation results on SAR images is only based on visual inspection. The Gaussian parameter of the spectral clustering method is set to 8, and other experimental settings are same with that of the previous subsection.

Fig. 5(a) is a section of SAR image over the city of Ottawa provided by the RADARSAT SAR sensor, which includes two types of ground objects of water (dark) and vacancy area (gray). Fig. 5(b)–(d), and (i) respectively show the segmentations of K-means, mean shift (with $(h_s, h_r, W) = (10, 8, 300)$), FLICM, and SPP, which have many spots. Although the segmentation results of MRF+LS and IENCut respectively shown in Fig. 5(e) and (f) have good regional consistency, a small number of pixels are misclassified. For the segmentations of KNN, LLE, L1, and RS_MLAP, some pixels are misclassified in the top left area of Fig. 5(g), (h), (j), and (k). By contrast, Fig. 5(l) obtained by RS_ESR has a favorable visual effect.

The second SAR image is a Ku-band SAR image with 3-m spatial resolution of the China Lake Airport, California, as shown in Fig. 6(a). This SAR image includes three types of ground objects of runway (dark), vacancy (gray), and airport buildings (bright). Fig. 6(b), (d), (f)–(j) have many misclassified pixels in vacancy areas, which show the segmentation results of K-means, FLICM, IENCut, KNN, LLE, SPP, and L1 respectively. The segmentation of mean shift with $(h_s, h_r, W) = (17, 10, 10)$ has a good visual effect, but it actually contains 103 categories, as shown in Fig. 6(c). In the segmentation result of MRF+LS shown in Fig. 6(e), some pixels are misclassified. The segmentation result obtained by RS_MLAP is shown in Fig. 6(k), which confuses the runway and airport buildings regions. Compared with other methods, the segmentation of RS_ESR shown in Fig. 6(l) distinguishes three classes and has good regional consistency.

5.4. Semi-supervised classification

The performance of RS_ESR based semi-supervised classification is evaluated by four set of experiments. Table 4 shows the semi-supervised classification results and the running times of KNN, LLE, SPP, L1, RS_MLAP, and RS_ESR on AR facial, MNIST handwritten digits and Cursive Character Challenge (C-Cube) [50] datasets. The AR

facial database includes 120 classes, and each class comprises 14 images, and each sample has 2000 dimension. In this experiment, we only use front 20 classes. Simultaneously, we utilize 100 samples of each class in MNIST handwritten digits dataset, and the number of dimension of each sample is 784. The C-Cube dataset contains capital data (C-CubeC) and lowercase data (C-CubeL). Similarly, we select randomly 100 samples of each class in C-CubeC and C-CubeL datasets (if the number of samples belonging to a class is smaller than 100, all samples from this class are used), and the number of dimension of each sample is 34 [48].

In addition, thirty percent of samples of each class are labeled, which is used to construct the initial labeled matrix F in Algorithm 5. In the semi-supervised classification experiments, the number of adjacent samples in the KNN and LLE methods is set to 10. Meanwhile, we set the number of random subspace q to 20 and dimensionality of each random subspace p to 100 for AR facial and MNIST handwritten digits. For C-CubeC and C-CubeL datasets, we set the two parameters to $q = 3$ and $p = 10$ respectively.

As you can see, the accuracy of RS_ESR is highest in Table 4. Moreover, though the runtimes of RS_MLAP and RS_ESR are longer than KNN and LLE, they are shorter than SPP and L1. It indicates that the random subspace method is effective for high dimensional data. By contrast, RS_ESR run quicker than RS_MLAP.

5.5. Discussion

In the RS_ESR based data clustering and image segmentation, the proposed RS_ESR method combines random subspace, sparse representation with spectral clustering methods. Which method has contributed most to the final result? This problem will be discussed in this subsection. Take the 4-class artificial image Fig. 2(a) as an example. Fig. 7 shows the segmentation results of spectral clustering, sparse representation based spectral clustering, RS_ESR algorithm on Fig. 2(a). As can be seen from Fig. 7, the segmentation of spectral clustering shown in Fig. 7(a) confuses many pixels with different classes. Fig. 7(b) distinguishes four classes, which shows the segmentation result of sparse representation based spectral clustering. It means that the sparse representation method has good category distinguishing performance and plays an important role in RS_ESR based clustering. Moreover, Fig. 7(c) obtained by RS_ESR has higher accuracy than Fig. 7(b), which elucidates that the usage of the random subspace method effectively improves the

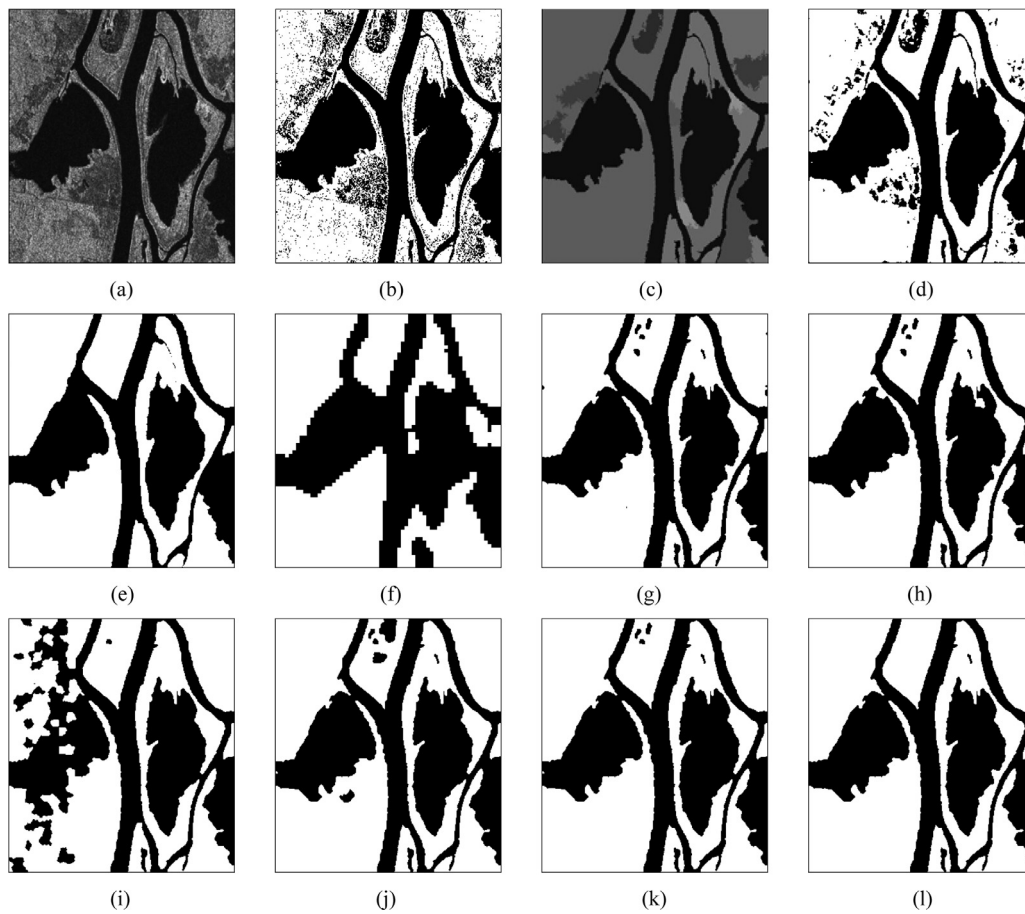


Fig. 5. Segmentation results on the SAR image of Ottawa (image size: 352×313). (a) Original image, (b) *K*-means, (c) Mean shift, (d) FLICM, (e) MRF+LS, (f) IENCut, (g) KNN, (h) LLE, (i) SPP, (j) L1, (k) RS_MLAP, (l) RS_ESR.

Table 4

Semi-supervised classification results and runtime (second) of six algorithms for different datasets.

		KNN	LLE	SPP	L1	RS_MLAP	RS_ESR
AR Facial Dataset	Accuracy	0.2286	0.3214	0.1542	0.4214	0.7571	0.8071
	Time	0.0821	0.036	110.4455	78.3494	60.8766	55.5955
MNIST Database of Handwritten Digits	Accuracy	0.272	0.456	0.46	0.484	0.756	0.844
	Time	0.0485	0.0317	483.8927	470.5629	388.6959	260.1916
C-Cube(C)	Accuracy	0.4851	0.5014	0.3538	0.4039	0.749	0.83
	Time	0.4345	0.5829	1128.295	1032.546	938.6681	612.2666
C-Cube(S)	Accuracy	0.4238	0.4979	0.3638	0.4322	0.5496	0.6238
	Time	0.0368	0.1015	1058.4273	931.9329	875.6390	586.5383

Algorithm 1 Random subspace method.

Input: a data set $X = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n | \mathbf{x}_i \in \mathbb{R}^d, i = 1, 2, \dots, n\}$, the number of random subspace q , the dimensionality of random subspace p ($p < d$).

1. Generate randomly q binary vectors $\mathbf{r}^t \in \mathbb{R}^d$ ($t = 1, \dots, q$), in which the j th element $r_j^t = \{0, 1\}$ ($j = 1, \dots, d$), and $\mathbf{r}^t$ satisfies the constraint $\sum_{j=1}^d r_j^t = p$.

2. Generate q random subspace sets as follows:

$$S^t = X(\mathbf{r}^t), \quad (3)$$

where $S^t = \{\mathbf{s}_1^t, \mathbf{s}_2^t, \dots, \mathbf{s}_n^t | \mathbf{s}_i^t \in \mathbb{R}^p\}$ is t th random subspace set, $\mathbf{s}_i^t$ includes the elements of $\mathbf{x}_i$ corresponding to $r_j^t = 1$.

Output: q random subspace sets $S^1, \dots, S^t, \dots, S^q$.

category distinguishing the performance of sparse representation based spectral clustering.

Furthermore, the proposed RS_ESR method includes two important parameters: number of random subspace q and dimensionality of each random subspace p . Taking the 2-class artificial image Fig. 1(a) as an example, the influence of different parameters for the segmentation results is shown in Fig. 8. It can be seen from Fig. 8 that the segmentation accuracy increases with the increasing

of q and p when $q < 11$ and $p < 35$. That is because more discriminative information is selected as the number of random subspace q and dimensionality of each random subspace p are larger. However, when $q > 10$ or $p > 30$, the segmentation accuracy slowly decreases in Fig. 8. It reflects that some redundant information is selected when q and p are too large, which leads to reduced segmentation accuracy. Thus the selection of appropriate parameters is very important to obtain ideal segmentation result.

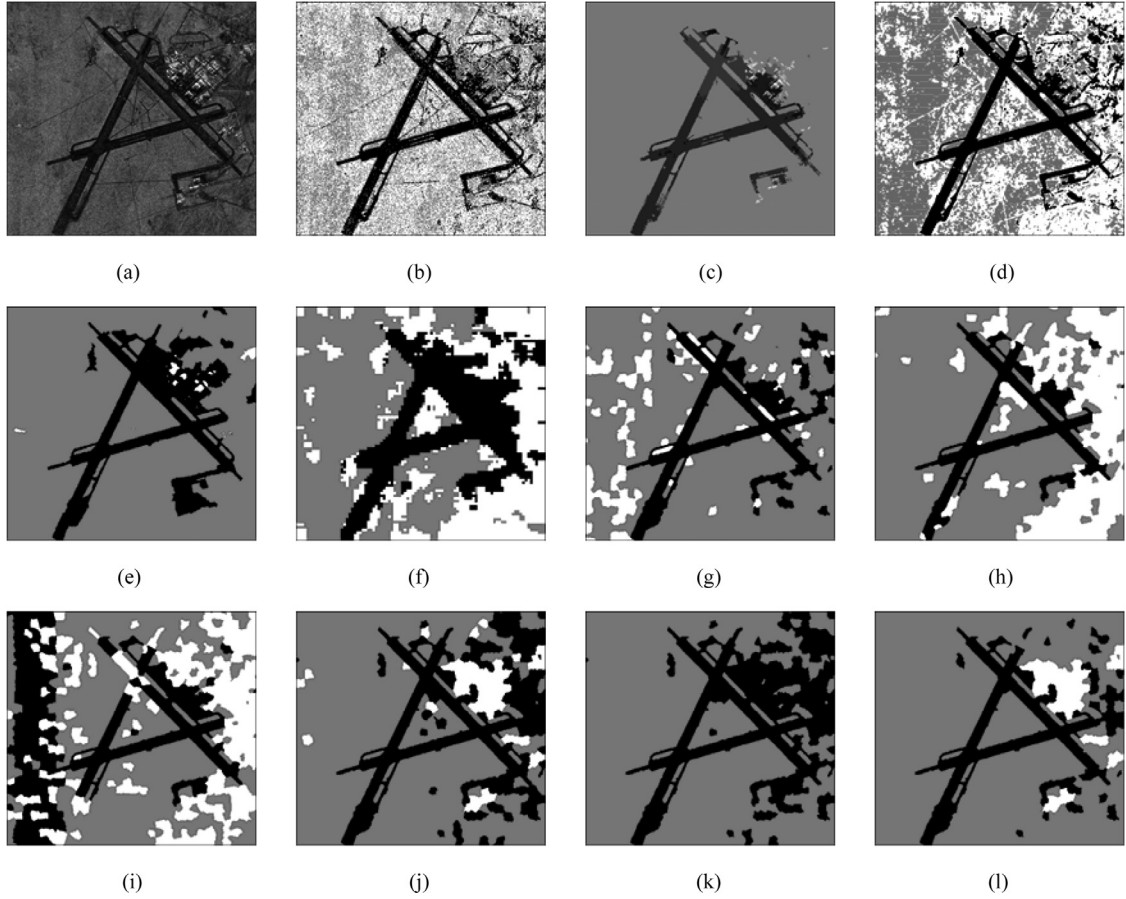


Fig. 6. Segmentation results on the SAR image of China Lake Airport (446 × 475 pixels). (a) Original image, (b) *K*-means, (c) Mean shift, (d) FLICM, (e) MRF+LS, (f) IENCut, (g) KNN, (h) LLE, (i) SPP, (j) L1, (k) RS_MLAP, (l) RS_ESR.

Algorithm 2 Solving the random subspace based joint sparse representation model by ADMM.

Input: multiple random subspace sets $S^1, \dots, S^t, \dots, S^q$, parameters α, β .

1. Initialize convergence threshold $\eta = 10^{-5}$, penalty parameter $\mu = 10^{-6}$ and $\theta = 1.1$, $Z^t = K^t = L^t = U^t = V^t = 0$.

2. Update $K^1, \dots, K^t, \dots, K^q$

$$\arg \min_{K^t} \frac{1}{\mu} \|K^t\|_1 + \frac{1}{2} \|K^t - (Z^t + \frac{U^t}{\mu})\|_F^2 \quad (5)$$

3. Update $L^1, \dots, L^t, \dots, L^q$, $L^t = ((S^t)^T S^t + \frac{\mu}{2\alpha} I)^{-1} ((S^t)^T S^t + \frac{\mu}{2\alpha} Z^t + \frac{1}{2\alpha} V^t)$, where I is an $n \times n$ identity matrix and T denotes transpose of a matrix.

4. Update Z

$$\arg \min_Z \frac{\beta}{\mu} \|Z\|_{2,1} + \frac{1}{2} \|Z - A\|_F^2 \quad (6)$$

$$\text{where } A = \begin{bmatrix} B_{11}^1 & B_{12}^1 & \dots & B_{1n}^1 \\ B_{21}^1 & B_{22}^1 & \dots & B_{2n}^1 \\ \vdots & \vdots & \ddots & \vdots \\ B_{11}^q & B_{12}^q & \dots & B_{1n}^q \end{bmatrix}, \text{ and } B^t = (K^t + L^t - (U^t + V^t)/\mu)/2.$$

5. Update the multipliers $U^t = U^t + \mu(Z^t - K^t)$, $V^t = V^t + \mu(Z^t - L^t)$.

6. Update the parameters $\mu = \min(\theta\mu, 10^{10})$.

7. If $\|Z^t - K^t\|_\infty < \eta$ and $\|Z^t - L^t\|_\infty < \eta$ finish the solution, otherwise return the step 2.

Output: q sparse representations $Z^1, \dots, Z^t, \dots, Z^q$.

Algorithm 3 Random Subspace Based Ensemble Sparse Representation (RS_ESR).

Input: data set X .

1. Generate multiple random subspace sets $S^1, \dots, S^t, \dots, S^q$ by Algorithm 1.

2. Calculate sparse representations $Z^1, \dots, Z^t, \dots, Z^q$ in each random subspace by Algorithm 2.

3. Integrate sparse representations $Z^1, \dots, Z^t, \dots, Z^q$ into an ensemble sparse representation E by (7).

Output: ensemble sparse representation E .

Algorithm 4 Spectral clustering based on RS_ESR.

Input: the $n \times n$ similarity matrix E , the number of clusters c .

1. Construct Laplacian matrix $L = W^{-\frac{1}{2}} E W^{-\frac{1}{2}}$, where $W = \{W_{ii} | 1 \leq i \leq n\}$ is a diagonal matrix where W_{ii} is the sum of E 's i -th row.

2. Find eigenvectors $g_1, g_2, \dots, g_c$ of L corresponding to c largest eigenvalues, and form the matrix $G = \{g_1, g_2, \dots, g_c\}$.

3. View each row of G as a sample, and use K-means algorithm to cluster them to get the final category vector.

Output: category vector of n samples.

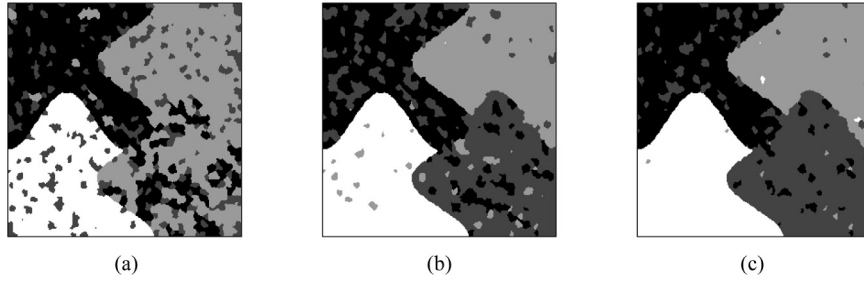


Fig. 7. Effects of different technologies. (a) Spectral clustering (accuracy: 0.6742), (b) Sparse representation based spectral clustering (accuracy: 0.859), (c) RS_ESR (accuracy: 0.9325).

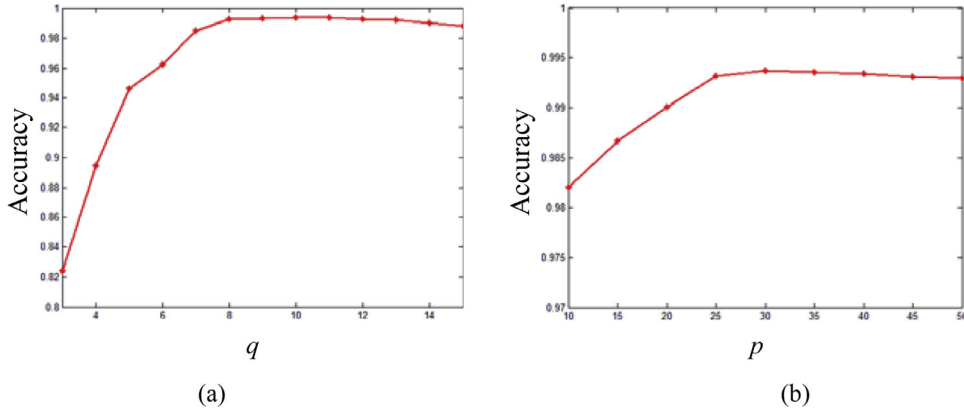


Fig. 8. The influence of different parameters on the RS_ESR based segmentation results. (a) The influence of the parameter q ($p = 30$). (b) The influence of the parameter p ($q = 10$).

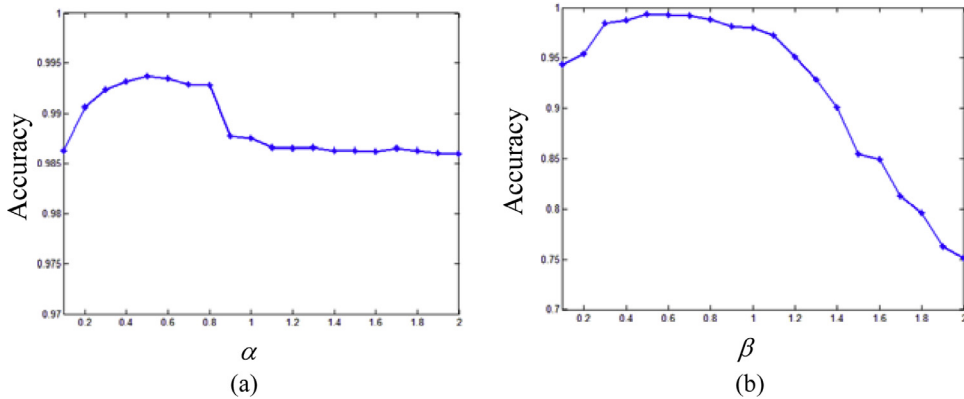


Fig. 9. The influence of different parameters for the segmentation results. (a) The influence of the parameter α ($\beta = 0.5$). (b) The influence of the parameter β ($\alpha = 0.5$).

Algorithm 5 Semi-supervised classification based on RS_ESR.

Input: the $n \times n$ similarity matrix E , a $n \times c$ (where n is number of samples and c is number of classes) initially labeled 0–1 matrix F ($F_{ia} = 1$ ($1 \leq i \leq n$, $1 \leq a \leq c$) if i -th sample is labeled as a -th class, otherwise $F_{ia} = 0$; meanwhile, since a small number of samples are labeled, the number of elements of values of 1 in F is much less than n), parameter $\lambda \in (0, 1)$.

1. Calculate final label matrix $\tilde{F} = (1 - \lambda)(I - \lambda E)^{-1}F$, where I is an $n \times n$ identity matrix.
2. Assign i -th sample to the category $\arg \max_{1 \leq a \leq c} \tilde{F}_{ia}$.

Output: category vector of n samples.

Finally, the solution of the objective function in (4) can be affected by two adjusting parameters α and β . Taking the 2-class synthetic image Fig. 1(a) as an example, the influence of different parameters on the segmentation results is shown in Fig. 9. As can be seen from Fig. 9(a), the curve is basically stable after a por-

tion of fluctuations. When $\alpha < 0.5$, the segmentation accuracy increases with the increasing of α in Fig. 9(a), which reflects that the fidelity term in (4) plays an important role in random subspace based joint sparse representation model. Similarly, the segmentation accuracy increases with the increasing of β when $\beta < 0.5$, as shown in Fig. 9(b). It indicates that the joint sparse representation term effectively improves the performance of the model. However, when α and β are too large, the accuracies gradually decrease in Fig. 9. That is, the impact of the latter two terms in (4) cannot be hugely magnified, since the first item of (4) also plays a not negligible role.

6. Conclusion

In this paper, a novel approach called random subspace based ensemble sparse representation (RS_ESR) was presented. Random subspace method randomly and repeatedly extracts a part from

original high-dimensional data to get multiple lower-dimensional samples. It has been proved that the probability that discriminative information of original data is not selected in multiple random subspaces is very low. Thus the random subspace algorithm can achieve the goal of dimensionality reduction, and also reserves effective information in original data. Then sparse representations of samples in multiple random subspaces are obtained by solving a joint sparse representation model based on random subspace. Afterwards, by a simple operation, these sparse representations acquired in random subspaces are integrated into an ensemble sparse representation, namely RS_ESR. Finally, RS_ESR is combined with specific tasks in machine learning. Considering the length of this paper, RS_ESR based spectral clustering and semi-supervised classification algorithms were briefly introduced. Experiments of RS_ESR on various datasets and images show that the performance of RS_ESR outperforms the relevant traditional methods.

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