

11th CIRP Conference on Industrial Product-Service Systems

Self-weighted Multi-view Subspace Clustering With Low Rank Tensor Constraint

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Abstract

Manufacturing process is making a difference in the Product-Service Systems (PSS). In the manufacturing process, the real-time high-dimensional data will be generated by the production line. In the field of high-dimensional data analysis, a very important analysis method is high-dimensional data clustering, and subspace clustering is one of the effective methods of high-dimensional data clustering. Clustering is an important topic in data mining. In real world, the data represented with multi view information makes the clustering algorithms more difficult. In this paper, we explore an improved multi-view clustering algorithm. Unlike the conventional multi-view subspace clustering methods that usually average the weight of views, the proposed algorithm simultaneously takes into account the expression and weight of each view. Specifically, every view is re-represented with the complementary information from multiple views by low rank tensor, and the views are fused by exploring a Laplacian rank constrained graph. With the complementary information from multiple views and the different weights of the views, the proposed algorithm is improved. Experimental results on various real-world datasets demonstrate the effectiveness of our research.

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Peer-review under responsibility of the scientific committee of the 11th CIRP Conference on Industrial Product-Service Systems

Keywords: multi-view, low-rank tensor, view-weighted

1. Introduction

With the rapid development of manufacturing industry and related disciplines, the massive data are generated in the manufacturing process. Therefore, for the object of production process, each factor data in the production process is abstracted into high-dimensional data. While effectively storing these massive high-dimensional data, it is necessary to effectively analyze these data, and then use the analysis conclusions to assist decision-making and guide production. In real world applications, the same thing can be described in different ways or from different perspectives, which constitute multi-views of the thing. More and more data can be represented in a multi-view fashion. For example, a text is able to be represented in multiple language; The web, which can be described by either the hyperlinks between web pages or the word occurring in webs [1], are represented as multi-view data. In general, the multi-view representation can capture the abundant and

complementary information from multi data sources or different perspectives.

As we mentioned earlier, owing to rich and complementary information of the multi-view representation, utilizing the multi-view will boost the performance of the clustering task.

In recent years, the complementary information of the multi-view representation have been utilized to boost the performance of the clustering task. The multi-view clustering [2,3] aimed to integrate different and multiple features into a consolidated framework. Much progress has made to develop effective multi-view clustering, which can be divided into three categories: co-training, multi-kernel learning and subspace clustering. The paradigm of co-training was first proposed in [4], whose main idea is that the information of views is used as the constraint in the process of training, and the training iterates until the clustering results of multiple views are consistent. The co-regularization framework was introduced by A. Kumar and H. Daum'e [5]. Multi-kernel learning [6] learns a predefined

kernel correspond to each view, and combines these kernels to fuse the views. In the algorithm [7], the weights of the kernel are computed and used to combine the views. The method in [8] was proposed to fuse the probabilistic transition matrix of all views, and the learned fusion of transition matrix was used as the input of the spectral clustering. In practical problems, high dimensional data usually resides in a low-dimensional manifold. In order to explore these high-dimensional data, subspace clustering is proposed, which segments different high dimensional data into their corresponding subspace. Subspace clustering has gained considerable attentions in dealing with multi-view clustering. The existing subspace clustering algorithms can roughly be categorized into five categories [9]: iterative [10], algebraic [11,12], statistical [13,14], matrix Factorization-based [15] and spectral clustering-based [4]. Among them, the spectral clustering-based methods are extremely popular and efficient, which have received a lot of attentions. Previous spectral clustering-based methods address the problem by two following steps. Firstly, an affinity matrix is learned from the multi-view data. Secondly, the affinity matrix is applied as the input of spectral clustering. Sparse Subspace Clustering (SSC) [16,17] and Low-Rank Clustering (LRR) [18,19], are both well-known approaches. Compared with other subspace clustering algorithms, sparse and low-rank has the advantage on dealing with noise. These methods are both based on the self-expressiveness model, in which each data can be expressed as a linear combination of other data. Based on the sparse and low-rank representation, C. Zhang et al. [20] proposed a low rank constraint tensor subspace clustering (LT-MSC), which integrates all subspace representation by tensor. This method performs well in capturing high order information. These subspace clustering methods assume that all the views have an equal contribution on clustering. However, this strategy is unreasonable. To discriminate the significance of different views in multi-view data multi-view kernel k-means (MVKKM) algorithm [7] is proposed to assign a weight for each view according to the contribution of the view on the clustering result and combine the kernels derived from the weighted views together. The method of Self-weighted with Multiple Graphs (SWMC) [21] was also proposed to use a Laplacian rank constraint graph [22], which directly assigned the cluster label to data without any postprocessing like K-means in spectral clustering.

As mentioned above, the multi-view clustering has been become a challenge task. In order to simultaneously capture high order relationships among multiple views, assign the different weights to different views, and without any postprocessing like K-means in spectral clustering. we explore an improved multi-view clustering algorithm. In the improved algorithm, the low rank constraint tensor is adopted to capture high order relationships among multiple views, which is used as the initial graph of the method in SWMC [21] that learns a good self-weighted similarity matrix. To demonstrate the advantages of the proposed algorithm, experiments have been conducted on Coil-20 dataset, the Extended YaleB dataset, the ETH-80 dataset, the Yale face dataset and the practical patent image dataset. The results of the experiments on these datasets demonstrate the effectiveness of our proposed algorithm.

2. The Proposed Algorithm

In this section, a brief introduction of low rank tensor constraint subspace clustering [20] is given since it is the base of our proposed algorithm. Then the Self-weighted Multi-view Subspace Clustering With Low Rank Tensor Constraint Algorithm and the implement of optimization are proposed, respectively

2.1. Low tensor constraint-based multi-view learning description

Suppose that there are N samples which belong to k clusters. In subspace clustering, the subspace representation matrix is usually used to construct the similarity matrix. However, these methods have the common disadvantage that ignores the intrinsic correlations among different views. Tensor is higher order arrays, which is the generalization of the matrix. Tensor can capture the high order correlations underlying multi-view data. The low rank tensor constraint multi-view clustering has been proposed in [20]. The target functions can be formulated as follows:

$$\min_{Z^v, E^v} \|Z\|_* + \lambda \|E\|_{2,1}$$

$$s. t. X^v = X^v Z^v + E^v, v = 1, 2, \dots, V, E = [E^1, E^2, \dots, E^V],$$

$$Z = \phi(Z^{(1)}, Z^{(2)}, \dots, Z^{(V)}) \quad (1)$$

Where $\phi(\cdot)$ Denotes the symbol constructing the tensor Z , which fusing the different representation Z^v to a 3-order tensor. $Z^v = [z_1^v, z_2^v, \dots, z_N^v] \in \mathbb{R}^{N \times N}$. $E = [E^1, E^2, \dots, E^V]$ means concatenating together with the reconstruction error of each columns. $\|\cdot\|_{2,1}$ is the $\ell_{2,1}$ -norm, which encourages the columns of E to be zero.

2.2. Self-weighted multi-view subspace clustering with low rank tensor constraint algorithm

In this paper, a self-weighted multi-view subspace clustering with low rank tensor constraint algorithm (SW-LT) is proposed to deal with the multi-view clustering problem. Given N samples with V multi-view. The data can be represented by the matrix $X^{(i)} = [x_1, x_2, \dots, x_N] \in \mathbb{R}^{D \times N}, i = 1 \dots V$. Each column of the matrix is D dimensional sample vector. To cluster the data into subspace representation matrix $Z^i, i = 1 \dots V$, and assign the weight by capturing the high order information, we integrate the framework of the low rank tensor constraint frame proposed in [20] and the self-weighted proposed in [21]. The objective function of the framework is formulated as:

$$\min_{S_{ij} \geq 0, S_i 1_n = 1, \text{rank}(L_S) = n-k, Z^v, E^v} \sum_{v=1}^V \|S - \sum_{v=1}^V (\mathcal{R}(Z^v) + \lambda_v L(X^v, X^v Z^v))\|_F^2$$

$$s. t. X^v = X^v Z^v + E^v, v = 1, 2, \dots, V,$$

$$E = [E^1, E^2, \dots, E^V], S \in \mathbb{R}^{n \times n} \quad (2)$$

Where X^v and Z^v denote the data matrix and the subspace representation matrix, respectively. z_i^v means the learned subspace representation of i -th samples in v -th view, and E^v is the reconstruction error matrix, $L(\cdot, \cdot)$ being the loss function. $R(\cdot)$ is the regularizer and λ is the hyperparameter, which controls the loss penalty for the v -th view. S denotes the target similarity matrix learned by the function, which is each row sums up to 1.

2.3. Optimization and algorithm

This section gives the optimization details for the proposed framework. It is not easy to solve the problem Eq. (2). In this section, we divide Eq. (2) into two dependent parts as Eq. (3) and Eq. (4). Thus, the formulated objective can be formulated as:

$$\min_{Z^v, E^v} \sum_{v=1}^V (\mathcal{R}(Z^v) + \lambda_v L(X^v, X^v Z^v)) \quad (3)$$

$$s. t. X^v = X^v Z^v + E^v, v = 1, 2, \dots, V$$

$$\min_{S_{ij} \geq 0, S_i \mathbf{1}_n = 1, \text{rank}(L_s) = n-k, Z^v, E^v} \sum_{v=1}^V \|S - SM^v\|_F^2 \quad (4)$$

We can solve Eq. (3) in a manner of Augmented Lagrange Alternating Direction Minimization (AL-ADM) algorithm. The optimization in Eq. (3) is specified in LT-MSC. According to section 2.1, we know the method of low tensor constraint proposed by C. Zhang et al. in [20] has the clearly advantages in multi-view clustering. However, the LT-MSC ignores that the different views have different contribution on clustering task. We use the similarities constructed by the subspace clustering representations as the initial similarity in SWMC [21]. According to Eq. (3) we can obtain the combined subspace representation matrix corresponding to different view, SM^v is given as the following:

$$SM^v = \frac{(z^{(v)} + z^{(v)T})}{2}, v = 1 \dots V \quad (5)$$

Eq. (4) can be re-written as Lagrange function:

$$\min_S \sum_{v=1}^m \|S - SM^v\|_F^2 + \mathcal{G}(\Lambda, S) \quad (6)$$

Λ denotes the Lagrange multiplier. $\mathcal{G}(\Lambda, S)$ is a proxy to constraint to S . Taking the derivative of Eq. (6) and setting it to zero, we have:

$$\sum_{v=1}^m w^v \frac{\partial \|S - SM^v\|_F^2}{\partial S} + \frac{\partial \mathcal{G}(\Lambda, S)}{\partial S} = 0 \quad (7)$$

Where w^v is given as the following:

$$w^v = \frac{1}{2\|S - SM^v\|_F^2} \quad (8)$$

When w^v is fixed, Eq. (7) can be taken place in the solution to the following problem:

$$\min_{S_{ij} \geq 0, S_i \mathbf{1}_n = 1, \text{rank}(L_s) = n-k, Z^v, E^v} \sum_{v=1}^V w^v \|S - SM^v\|_F^2 \quad (9)$$

The calculated S from Eq. (9) can be used to update w^v by Eq. (8). This inspires us to solve the Eq. (4) by alternately optimizing S and w^v iteratively. Let $\varepsilon_i(L_s)$ denote the i -th smallest eigenvalue of L_s . Note that $\varepsilon_i(L_s) \geq 0$ because L_s is positive semidefinite. The problem (4) is equivalent to the following problem for a large enough value of λ :

$$\min_{S_{ij} \geq 0, S_i \mathbf{1}_n = 1} \sum_{v=1}^V w^v \|S - SM^v\|_F^2 + 2\lambda \sum_{i=1}^k \varepsilon_i(L_s) \quad (10)$$

According to Ky Fan's Theorem [23], we have:

$$\sum_{i=1}^k \varepsilon_i(L_s) = \min_{F \in \mathbb{R}^{n \times k}, F^T F = I} \text{Tr}(F^T L_s F) \quad (11)$$

Therefore, the problem (10) is further equivalent to the following problem:

$$\min_{S, F} \sum_{v=1}^V w^v \|S - SM^v\|_F^2 + 2\lambda \text{Tr}(F^T L_s F) \quad (12)$$

$$s. t. S_i \mathbf{1}_n = 1, S_{ij} \geq 0, F \in \mathbb{R}^{n \times k}, F^T F = I$$

When S is fixed, the problem (12) becomes:

$$\min_{F \in \mathbb{R}^{n \times k}, F^T F = I} \text{Tr}(F^T L_s F) \quad (13)$$

When F is fixed, the problem (12) becomes:

$$\min_{S_{ij} \geq 0, S_i \mathbf{1}_n = 1} \sum_{v=1}^V w^v \sum_{i,j=1}^n (s_{ij} - sm_{ij}^v)^2 + \lambda \sum_{i,j=1}^n \|f_i - f_j\|_2^2 s_{ij} \quad (14)$$

Note that the problem (12) is independent for different i . The following problem can be separately solved for each i :

$$\min_{S_{ij} \geq 0, S_i \mathbf{1}_n = 1} \sum_{j=1}^n \sum_{v=1}^V w^v (s_{ij} - sm_{ij}^v)^2 + \lambda \sum_{j=1}^n \|f_i - f_j\|_2^2 s_{ij} \quad (15)$$

The above problem can be solved by the efficient iterative algorithm by Huang, Nie, and Huang [24]. We summarize the process as Algorithm 1.

Algorithm 1 Self-weighted Multi-view Subspace Clustering With Low Rank Tensor Constraint Algorithm

Input: multi-view data matrix: $X^{(1)}, X^{(2)}, \dots, X^{(V)}$, and the cluster number k :

Initialize: $Z^{(1)} = 0, \dots, Z^{(V)} = 0$; $E^{(1)} = 0, \dots, E^{(V)} = 0$; the weight for each view (e.g., $w^v = \frac{1}{V}$).

Calculate $z^{(v)}$ by the algorithm by solving the problem (1) by AL-ADM in [20].

Calculate SM^v by solving the problem (5).

repeat

1. Calculate S by solving the problem (9).

2. Update w^v by using Eq. (8).

until converge

Output: $S \in \mathbb{R}^{n \times n}$ with exactly k connected components

3. Experiment and evaluation

In this section, five real-world datasets are used to evaluate our algorithm. Some images from the datasets are shown in Fig 1. The brief description is given in Table 1.

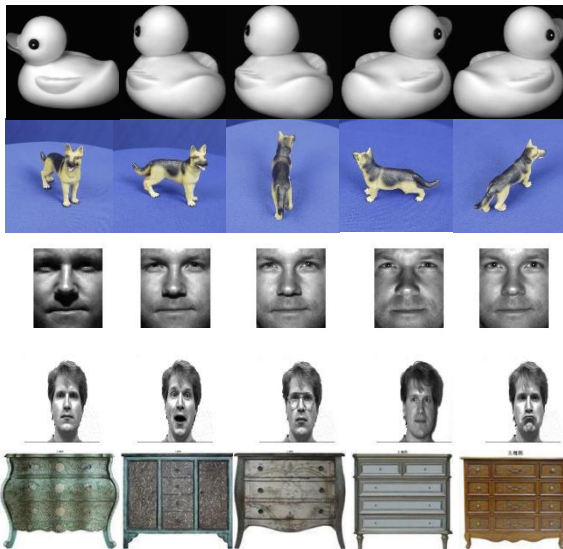


Fig 1. Example images of the five datasets used in this paper (the rows from top to bottom correspond to Coil-20, Eth-80, Extended-YaleB, Yale face and Patent image, respectively).

Table 1. Statistics of five datasets

Feature	COIL-20	ETH-80	Extended-YaleB	Yale face	Patent image
HOG	324	324	324	324	324
LBP	236	236	236	236	236
Gabor	320	320	320	320	320
#of data	1440	328	640	165	400
#of class	20	8	10	15	4

Our method is compared with some baseline algorithms, which include 2 single view and 4 multi-view as follows:

- SPC [25] (best): the method is the standard spectral clustering algorithm with the best view.
- LLR [18] (best): the method constructs similarity matrix with the best single view in low rank constraint subspace clustering.
- Co-Reg SPC [8]: the method co-regularizes the clustering to enforce corresponding data points with the

same clustering.

- Co-Training SPC [4]: the method uses the manner of the co-training in the spectral clustering framework.
- LT-MS [20]: the method uses the low rank tensor constraint to construct the similarity matrix as the input of spectral clustering.
- SWMC [21]: the method is a multi-view clustering with totally self-weighted.

Six evaluation metrics are adopted to evaluate the performances: Normalized Mutual Information (NMI), Accuracy (ACC), Adjusted Rand index (AR), F-score, Precision and Recall. We report the detailed clustering results on five datasets in Table 2-6:

Table 2. Results on Coil-20

Method	NMI	ACC	AR	F	P	R
SPCbest	0.825	0.693	0.627	0.648	0.588	0.723
LRRbest	0.829	0.763	0.694	0.710	0.670	0.756
Co-Reg	0.774	0.626	0.572	0.594	0.560	0.633
Co-Training	0.825	0.695	0.647	0.647	0.623	0.718
LTMSC	0.871	0.776	0.743	0.756	0.721	0.795
SWMC	0.918	0.781	0.778	0.790	0.691	0.922
SW-LT	0.928	0.804	0.807	0.817	0.768	0.873

Table 3. Results on ETH-80

Method	NMI	ACC	AR	F	P	R
SPCbest	0.335	0.359	0.385	0.283	0.277	0.288
LRRbest	0.565	0.530	0.381	0.461	0.437	0.481
Co-Reg	0.528	0.525	0.353	0.435	0.416	0.457
Co-Training	0.603	0.536	0.391	0.391	0.420	0.552
LTMSC	0.683	0.622	0.477	0.548	0.495	0.614
SWMC	0.611	0.521	0.341	0.451	0.323	0.769
SW-LT	0.718	0.561	0.378	0.484	0.346	0.801

Table 4. Results on Extended-YaleB

Method	NMI	ACC	AR	F	P	R
SPCbest	0.689	0.607	0.527	0.576	0.546	0.609
LRRbest	0.751	0.765	0.625	0.663	0.650	0.690
Co-Reg	0.658	0.611	0.502	0.553	0.531	0.586
Co-training	0.737	0.655	0.578	0.578	0.601	0.685
LTMSC	0.764	0.751	0.654	0.689	0.678	0.700
SWMC	0.810	0.679	0.640	0.681	0.588	0.808
SW-LT	0.841	0.828	0.676	0.711	0.658	0.772

Table 5. Results on Yale face

Method	NMI	ACC	AR	F	P	R
SPCbest	0.656	0.655	0.508	0.541	0.499	0.590
LRRbest	0.701	0.649	0.478	0.514	0.473	0.565
Co-reg	0.668	0.640	0.502	0.568	0.503	0.572
Co-training	0.672	0.630	0.566	0.535	0.544	0.630
LTMSC	0.763	0.727	0.738	0.621	0.597	0.647
SWMC	0.692	0.660	0.446	0.484	0.429	0.555
SW-LT	0.735	0.697	0.523	0.555	0.507	0.613

Table 6. Results on Patent image

Method	NMI	ACC	AR	F	P	R
SPCbest	0.560	0.797	0.553	0.664	0.661	0.667
LRRbest	0.780	0.917	0.791	0.843	0.838	0.848
Co-reg	0.878	0.957	0.890	0.918	0.905	0.913
Co-training	0.918	0.950	0.904	0.927	0.908	0.910
LTMSC	0.887	0.955	0.887	0.915	0.890	0.911
SWMC	0.850	0.823	0.767	0.834	0.763	0.936
SW-LT	0.947	0.895	0.872	0.895	0.904	0.914

Table 2-6 shows the clustering NMI, ACC, AR, F-score, Precision and Recall of the methods on five datasets respectively. As shown in the tables, our method can achieve better performance than the compared methods in most cases. It can also be observed that LT-MSC and SWMC usually obtains the good results, but sometimes fails. As we mentioned before, LT-MSC uses an equal weight, which ignores the contribution corresponding each view. The initialization of SWMC is an arbitrary similarity matrix and decrease the clustering accuracy if the initialized similarity matrix has low quality. Meanwhile, it is clear that combining the advantages of the low rank tensor constraint and self-weighted is helpful to better cluster the data with multi-views. In the Product-Service Systems, intelligent production platform will improve the quality of product. This improved multi view clustering can combine with mature agile development frameworks such as spring, developing a data analysis platform for manufacturing process and improving data analysis and decision-making efficiency. We use this algorithm as the core of the module of high-dimensional data clustering. This module will return an object to encapsulates view information and clustering results. When the controller receives this object and calls the component to locate the view passing the clustering information.

4. Conclusion

In this paper, we explore the clustering problem for multi-view data. An improved multi-view clustering algorithm has been proposed. The proposed algorithm follows the method of low rank tensor constraint to construct the similarity matrix of V views, and adopts it as the initialized similarity matrix of the self-weighted multi-view graph learning. The compared experiments have been conducted on five benchmark datasets and the experimental results demonstrate the effectiveness of our algorithm. In the future, we will further improve the weighting method and consider extending the framework to improve Product-Service Systems (PSS).

Acknowledgements

This research was supported by the National Natural Science Foundation of China (No. U1701266), the Natural Science Foundation of Guangdong Province (No. 2018A030313751), the Science and Technology Planning Project of Guangdong Province (No. 2014B050502014, No. 2017B090901056, No. 2018B030322016) and the Guangdong

Higher Education Engineering Technology Research Center for Big Data on Intellectual Property of Manufacturing.

References

- [1] Bickel S, Scheffer T. Multi-View Clustering[C]// IEEE International Conference on Data Mining. IEEE Computer Society, 2004.
- [2] Wei T, Lu Z, Dhillon I S. Clustering with Multiple Graphs[C]// Ninth IEEE International Conference on Data Mining. 2009.
- [3] De Sa V R. Spectral clustering with two views[C]//ICML workshop on learning with multiple views. 2005: 20-27.
- [4] Kumar A, Daumé III, H. A Co-training Approach for Multi-view Spectral Clustering[C]// International Conference on International Conference on Machine Learning. Omnipress, 2011.
- [5] Kumar A, Rai P, Daumé III, Hal. Co-regularized multi-view spectral clustering[C]// Proceedings of the 24th International Conference on Neural Information Processing Systems. Curran Associates Inc. 2011.
- [6] Cortes C, Mohri M, Rostamizadeh A. Learning non-linear combinations of kernels[C]// International Conference on Neural Information Processing Systems. Curran Associates Inc. 2009.
- [7] Tzortzis G, Likas A. Kernel-Based Weighted Multi-view Clustering[C]// Data Mining (ICDM), 2012 IEEE 12th International Conference on. IEEE, 2012.
- [8] Xia R, Pan Y, Du L, et al. Robust multi-view spectral clustering via low-rank and sparse decomposition[C]//Twenty-Eighth AAAI Conference on Artificial Intelligence. 2014.
- [9] Li C G, Vidal R. Structured Sparse Subspace Clustering: A Unified Optimization Framework[J]. 2015.
- [10] Bradley P S, Mangasarian O L. k-Plane Clustering[M]. Kluwer Academic Publishers, 2000.
- [11] Huang K, Ma Y, Vidal R. Minimum effective dimension for mixtures of subspaces: A robust GPCA algorithm and its applications[C]//Proceedings of the 2004 IEEE Computer Society Conference on Computer Vision and Pattern Recognition, 2004. CVPR 2004. IEEE, 2004, 2: II-II.
- [12] Ma Y, Yang A Y, Fossum D R. Estimation of Subspace Arrangements with Applications in Modeling and Segmenting Mixed Data[J]. SIAM Review, 2008, 50(3):413-458.
- [13] Leonardis A, Bischof H, Mayer J. Multiple eigenspaces[J]. Pattern Recognition, 2002, 35(11):2613-2627.
- [14] Yang A Y, Rao S R, Ma Y. Robust statistical estimation and segmentation of multiple subspaces[C]//2006 Conference on Computer Vision and Pattern Recognition Workshop (CVPRW'06). IEEE, 2006: 99-99.
- [15] Boulton T E, Brown L G. Factorization-based Segmentation of Motions[J]. Proc IEEE Workshop on Visual Motion, 1991:179 - 186.
- [16] Elhamifar E, Vidal R. Sparse subspace clustering[C]// IEEE Conference on Computer Vision & Pattern Recognition. 2009.
- [17] Elhamifar E, Vidal R. Sparse Subspace Clustering: Algorithm, Theory, and Applications[J]. IEEE Transactions on Pattern Analysis and Machine Intelligence, 2013, 35(11):2765-2781.
- [18] Liu G, Lin Z, Yan S, et al. Robust Recovery of Subspace Structures by Low-Rank Representation[J]. IEEE Transactions on Pattern Analysis and Machine Intelligence, 2013, 35(1):171-184.
- [19] Liu G, Lin Z, Yu Y. Robust subspace segmentation by low-rank representation[C]//Proceedings of the 27th international conference on machine learning (ICML-10). 2010: 663-670.
- [20] Zhang C, Fu H, Liu S, et al. Low-Rank Tensor Constrained Multiview Subspace Clustering[C]// ICCV 2015. IEEE Computer Society, 2015.
- [21] Nie F, Li J, Li X. Self-weighted Multiview Clustering with Multiple Graphs[C]//IJCAI. 2017: 2564-2570.
- [22] Nie F, Wang X, Jordan M I, et al. The constrained laplacian rank algorithm for graph-based clustering[C]//Thirtieth AAAI Conference on Artificial Intelligence. 2016.
- [23] Fan, K. On a Theorem of Weyl Concerning Eigenvalues of Linear Transformations: II[J]. Proceedings of the National Academy of Sciences, 1950, 36(1):31-35.
- [24] Huang J, Nie F, Huang H. A new simplex sparse learning model to measure data similarity for clustering[C]// International Conference on Artificial Intelligence. AAAI Press, 2015.
- [25] Zhang C, Fu H, Liu S, et al. Low-Rank Tensor Constrained Multiview Subspace Clustering[C]// ICCV 2015. IEEE Computer Society, 2015.